The results obtained yield quantitative estimates of the errors associated with replacement of the spatial temperature distributions of the medium and of the heat-flux density of their values averaged over the boundary section, which permits a well-founded approach to the selection of algorithms of the step-by-step modeling of the thermal regime of a system of bodies.

NOTATION

T, T_m, temperature of the body and the conditional medium; q, heat-flux density; λ, heat conductivity; α, heat-transfer coefficient; q_V, specific power of the heat sources; x, radius-vector; θ, relative error in computing the temperature under average boundary conditions; l_k, governing dimension of a section of the boundary Γ_k; x', y', relative coordinates; Bi, Biot criterion.

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APPROXIMATE SOLUTION OF THE STEFAN PROBLEM ON A SEGMENT

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The behavior of the temperature and the boundary near the stationary state are studied in the single-phase Stefan problem for certain types of thermal flux variations.

The perturbation of the stationary solution of the Stefan problem on a segment for small changes in the flux acting on the boundary was examined in [1, 2], where general expressions are obtained for the boundary and temperature for $0 \leqslant t < \infty$*. A detailed investigation of this approximate solution is quite important in applications but it is difficult for arbitrary flux perturbations. The most characteristic cases (step and sinusoidal thermal flux variation) are analyzed in detail in this paper, hence, asymptotic formulas are obtained for the solution for "small" and "large" times. The general solution of the problem under consideration is also simplified for slow and smooth flux changes.

The problem is formulated as follows. Find the classical solution of a system of equations with additional conditions

*The method used to obtain this solution was also applied in the two-phase problem [3].

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$$\frac{\partial v}{\partial \tau} = a^2 \frac{\partial^2 v}{\partial x^2},\ \tau > 0,\ 0 < x < s(\tau),\ a^2 = k/\rho c;\ -k\frac{\partial v}{\partial x}(s(\tau),\ \tau) + Q(\tau) = -\lambda\rho\frac{ds}{d\tau},\ \tau > 0,\ Q(\tau) \quad (1)$$

is a piecewise-continuous function, $s(0) = s_0 > 0$

$$v(x,\ 0) = T_0 + (T_m - T_0)x/s_0,\ 0 \leqslant x \leqslant s_0;\ v(0,\ \tau) = T_0,$$
$$v(s(\tau),\ \tau) = T_m;\ \tau > 0.$$

Here $T_0 \neq T_m$, $\lambda/c(T_m - T_0) = \beta \geqslant 0$. The system (1) describes the behavior of a heat-conducting flat layer with moving phase ($\lambda \neq 0$) or thermal ($\lambda = 0$) boundary $s(\tau)$.* If the acting thermal flux has the form $Q(\tau) = Q_0(1 + q(\tau))$, where $Q_0 = k(T_m - T_0)/s_0$, $\max\limits_{\tau\geqslant 0}|q(\tau)| \to 0$, then as is shown in [1, 2], the solution of the problem in an approximation linear in the flux variations is given by the formulas:

$$s(\tau) = s_0(1 - \mu(\tau)),\ v(x,\ \tau) = T_0 + (T_m - T_0)x/s_0 + w(x/s(\tau),\ \tau), \quad (2)$$

where

$$\mu(\tau) = \int_0^\tau K(\tau - \tau')q(\tau')d\tau',\ w(p,\ \tau) = \int_0^\tau L(p,\ \tau - \tau')q(\tau')d\tau', \quad (3)$$

and the following kernals are in the convolutions

$$K(\tau) = \frac{2}{\tau_0}\sum_{n=1}^{\infty}\frac{\exp(-b_n^2\tau/\tau_0)}{1 + \beta + \beta^2 b_n^2},\ \tau_0 = s_0^2/a^2; \quad (4)$$

$$L(p,\ \tau) = (T_m - T_0)\frac{2}{\tau_0}\sum_{n=1}^{\infty}\frac{\sin pb_n}{\sin b_n}\ \frac{\exp(-b_n^2\tau/\tau_0)}{1 + \beta + \beta^2 b_n^2}, \quad (5)$$

where b_n are positive roots of the equation $\operatorname{ctg} b_n = \beta b_n$.

Relying on these results, we study the behavior of the solution for fluxes of the simplest form.

Step Perturbation. Let $q(\tau) \equiv q = \text{const}$. Using (3)-(5), we obtain

$$\mu(\tau) = q\left[1 - 2\sum_{n=1}^{\infty}\frac{\exp(-b_n^2\tau/\tau_0)}{b_n^2(1 + \beta + \beta^2 b_n^2)}\right]\xrightarrow[\tau\to\infty]{} q, \quad (6)$$

$$w(p,\ \tau) = q(T_m - T_0)\left[p - 2\sum_{n=1}^{\infty}\frac{\sin pb_n}{\sin b_n}\ \frac{\exp(-b_n^2\tau/\tau_0)}{b_n^2(1 + \beta + \beta^2 b_n^2)}\right]\xrightarrow[\tau\to\infty]{} q(T_m - T_0)p. \quad (7)$$

Hence it is seen that the boundary emerges monotonically at a new stationary level. The temperature $v(x,\ \tau)$ (according to property b) in [2]), while also varying monotonically, tends to a new linear distribution as $\tau \to \infty$.

The asymptotic of the boundary for "small" times was studied in [1] and has the form

$$\mu(\tau) \cong q\left(1 - \frac{2}{\beta}\sqrt{\tau/\pi\tau_0}\right)\tau/\tau_m;\ \beta > 0,\ \tau_m = \beta\tau_0,\ 0 \leqslant \tau \ll \tau_0\min(1;\ \beta^2)$$

$$\mu(\tau) \cong 2q\sqrt{\tau/\pi\tau_0},\ \beta = 0,\ 0 < \tau \ll \tau_0.$$

The asymptotic of the solution for "large" times is obtained by retaining the highest term in the series (6) and (7). Extracting these terms, we write the solution in the form

$$\mu(\tau) = q\left[1 - 2\frac{\exp(-b_1^2\tau/\tau_0)}{b_1^2(1 + \beta + \beta^2 b_1^2)}(1 + \delta_1(\tau))\right], \quad (8)$$

$$w(p,\ \tau) = q(T_m - T_0)\left[p - 2\ \frac{\sin pb_1}{\sin b_1}\ \frac{\exp(-b_1^2\tau/\tau_0)}{b_1^2(1 + \beta + \beta^2 b_1^2)}(1 + \delta_2(p,\ \tau))\right]. \quad (9)$$

*This is a provisional classification since certain phase transitions can, as is known, have a zero latent heat.

Here the remainders are estimated as follows

$$0<\delta_1(\tau)<\frac{b_1^2}{b_2^2}\left(1+\frac{\tau_0^2}{2\pi^5\tau^2}\right)\exp\left(-\pi^2\frac{\tau}{\tau_0}\right),\ \tau>0;$$

$$|\delta_2(p,\ \tau)|<3\frac{b_1^2}{b_2^2}\left(1+\frac{\tau_0^2}{\pi^4\tau^2}\right)\exp\left(-\pi^2\frac{\tau}{\tau_0}\right);\ \tau>0,\ 0\leqslant p\leqslant 1.$$

Hence, the time of solution emergence in the exponential regime can be estimated. For instance, it is a quantity $\leqslant\tau_0/10$ for the boundary, where this estimate diminishes as β grows. Let us note that the remainder (8) can still be estimated independently of time:

$$0<\delta_1(\tau)<\frac{1}{2}(3\beta+1)^{-2},$$

from which it is clear that for a sufficiently large heat of transition ($\beta\geqslant 1$) the behavior of the boundary is close to the exponential on the whole time axis. The characteristic time of passage of the system into the new stationary state is determined from (8) and (9) and equals $\tau_0/b_1^2\geqslant\tau_0$; it grows as β increases.

Sinusoidal Perturbation. Let $q(\tau) = q\sin\omega\tau$; $\omega,\ q > 0$. Let us analyze the behavior of the boundary. Substituting into (4), we have

$$\mu(\tau)=q\,[g(\tau)+A\sin\omega\tau-B\cos\omega\tau];\ g(\tau),\ A,\ B>0,$$

where

$$g(\tau)=2\omega\tau_0\sum_{n=1}^{\infty}\frac{\exp(-b_n^2\tau/\tau_0)}{(1+\beta+\beta^2b_n^2)(b_n^4+\omega^2\tau_0^2)}\xrightarrow[\tau\to\infty]{}0. \qquad (10)$$

This monotonic function describes the transition regime

$$A=A_\beta(\omega)=2\sum_{n=1}^{\infty}\frac{b_n^2}{(1+\beta+\beta^2b_n^2)(b_n^4+\omega^2\tau_0^2)}\xrightarrow[\omega\to\infty]{}0, \qquad (11)$$

$$B=B_\beta(\omega)=2\omega\tau_0\sum_{n=1}^{\infty}\frac{1}{(1+\beta+\beta^2b_n^2)(b_n^4+\omega^2\tau_0^2)}\xrightarrow[\omega\to\infty]{}0. \qquad (12)$$

The steady-state regime can be written in the form

$$\mu(\tau)\underset{\tau\to\infty}{\cong}\bar{\mu}(\tau)\equiv qD\sin(\omega\tau-\varphi),$$

where

$$D=D_\beta(\omega)=\sqrt{A_\beta^2(\omega)+B_\beta^2(\omega)};\ \varphi=\varphi_\beta(\omega)=\operatorname{arctg}\frac{B_\beta(\omega)}{A_\beta(\omega)}.$$

This is a harmonic oscillation with flux perturbation frequency and phase delay $0<\varphi<\pi/2$. The build-up time for this regime is determined from the inequality $g(\tau)\ll D$ to satisfy which it is sufficient that $g(\tau)\ll\max(A;\ B)$. It follows from (10)-(12) that the last inequality is satisfied in every case by starting with a time on the order of several τ_0/b_1^2, while for sufficiently low frequencies ($\omega\tau_0\ll b_1^2$) it is valid even for $\tau = 0$. Therefore, the characteristic build-up time of the boundary oscillations does not exceed τ_0/b_1^2 and tends to zero as the frequency decreases.

Let us find the amplitude and phase of regular oscillations. We consider the Laplace transform of the solution

$$\mu(\tau)\risingdotseq M(r)=\frac{\omega q}{r^2+\omega^2}k(r),\ \text{where}\ k(r)\risingdotseq K(\tau)\ \text{(see [2])},$$

which has two singularities with highest real part (a first-order pole): $r_{1,2} = \pm i\omega$. In the neighborhood of each it is expanded in a series of such a form (the upper sign refers to r_1 and the lower to r_2):

$$M(r)=\sum_{n=-1}^{\infty}f_n^{\pm}(r\mp i\omega)^n,$$

where

$$f_{-1}^{\pm} = \operatorname{Res} M(\pm i\omega) = \frac{\omega q}{2r} k(r) \Big|_{r=\pm i\omega} = \pm q \frac{k(\pm i\omega)}{2i}.$$

Now applying the theorem of the asymptotic of the original at large times [4], we obtain

$$\bar{\mu}(\tau) = \exp(i\omega\tau) \sum_{n=-1}^{\infty} \frac{f_n^+}{\Gamma(-n)\tau^{n+1}} + \exp(-i\omega\tau) \sum_{n=-1}^{\infty} \frac{f_n^-}{\Gamma(-n)\tau^{n+1}} = q \frac{\exp(i\omega\tau) k(i\omega) - \exp(-i\omega\tau) k(-i\omega)}{2i}.$$

The kernal $K(\tau)$ is a real function, consequently, its transform possesses the conjugate property $k(r^*) = k^*(r)$. Hence

$$\bar{\mu}(\tau) = q \operatorname{Im}[\exp(i\omega\tau) k(i\omega)] = qD_\beta(\omega) \sin(\omega\tau - \varphi_\beta(\omega)),$$

where

$$D_\beta(\omega) = \frac{2}{\gamma} \frac{\operatorname{ch}\gamma - \cos\gamma}{\sqrt{(\operatorname{sh}\gamma + \sin\gamma)^2 + [\beta\gamma(\operatorname{ch}\gamma - \cos\gamma) + \operatorname{sh}\gamma - \sin\gamma]^2}}; \tag{13}$$

$$\varphi_\beta(\omega) = \operatorname{arctg} \frac{\beta\gamma(\operatorname{ch}\gamma - \cos\gamma) + \operatorname{sh}\gamma - \sin\gamma}{\operatorname{sh}\gamma + \sin\gamma}.$$

Here we introduced the notation $\gamma = \sqrt{2\omega\tau_0}$. We now analyze the dependences obtained.

1. Thermal boundary, $\beta = 0$:

$$D_0(\omega) = \sqrt{\frac{1}{\omega\tau_0} \frac{\operatorname{ch}\gamma - \cos\gamma}{\operatorname{ch}\gamma + \cos\gamma}}, \quad \varphi_0(\omega) = \operatorname{arctg} \frac{\operatorname{sh}\gamma - \sin\gamma}{\operatorname{sh}\gamma + \sin\gamma}.$$

It can be shown that $D_0(\omega)$ decreases monotonically with frequency. The phase is not a monotonic function of ω. It tends to a finite limit as the frequency grows $\varphi_0(\omega) \underset{\omega\to\infty}{\longrightarrow} \operatorname{arctg} 1 = \pi/4$, but intersects its asymptote at the points where $\gamma = \pi n$ ($n = 1, 2, \ldots$).

For "low" frequencies ($\omega\tau_0 \lesssim 1$)

$$D_0(\omega) \cong 1/\sqrt{1 + \omega^2\tau_0^2/6}, \quad \varphi_0(\omega) \cong \omega\tau_0/3.$$

For "high" frequencies ($\omega\tau_0 \gg 1$)

$$D_0(\omega) \cong 1/\sqrt{\omega\tau_0}, \quad \varphi_0(\omega) \cong \pi/4. \tag{14}$$

2. Phase boundary, $\beta > 0$. In this case the decrease in amplitude with frequency can also be shown. As follows from (13), the dependence in the parameter β is continuous and monotonic. Other conditions being equal, the higher the latent heat of transition of the material, the smaller the amplitude of the boundary oscillations and the higher the phase shift. Therefore, the curves of each of the parametric families $D_\beta(\omega)$ and $\varphi_\beta(\omega)$ are arranged continuously after each other, where the characteristics $D_0(\omega)$ and $\varphi_0(\omega)$ of the thermal boundary occupy an extreme location.

For "low" frequencies ($\omega\tau_0 \lesssim 1$)

$$D_\beta(\omega) \cong 1/\sqrt{1 + \omega^2\tau_0^2[(\beta + 1/3)^2 + 1/18]}, \quad \varphi_\beta(\omega) \cong \operatorname{arctg}[(\beta + 1/3)\omega\tau_0].$$

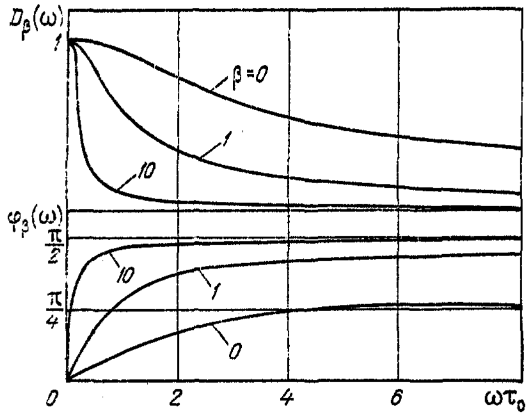
For "high" frequencies ($\omega\tau_0 \gg 1$)

$$D_\beta(\omega) \cong \sqrt{\frac{2}{\omega\tau_0}} \frac{1}{\sqrt{1 + (1 + \beta\sqrt{2\omega\tau_0})^2}} \cong (\text{for } \sqrt{\omega\tau_0} \gg 1/\beta) \cong 1/\omega\tau_m,$$

$$\varphi_\beta(\omega) \cong \operatorname{arctg}(1 + \beta\sqrt{2\omega\tau_0}) \underset{\omega\to\infty}{\longrightarrow} \pi/2. \tag{15}$$

Let us note certain properties of the characteristics (see sketch). All the AFC (amplitude—frequency characteristics) emerge parabolically from the maximum for low frequencies and by decreasing tend hyperbolically to the abscissa axis at high frequencies. All the PFC (phase—frequency characteristics) emerge linearly from the origin and tend to the asymptote as $\omega \to \infty$. However, the thermal boundary differs qualitatively from the phase by its behavior as $\omega \to \infty$ (compare (14) and (15)).

3. High Heat of Transition, $\beta \to \infty$. According to [2], the kernal $K(\tau)$ approximately equals the first term in the series (4) with a relative error of $\delta_1(\tau) < 1/2\beta$ ($\tau \geqslant 0$, $\beta > 0$). By finding that $b_1 \approx 1/\sqrt{\beta}$ for large β, we hence obtain a solution on the whole time axis



Sketch. Amplitude-frequency $D_\beta(\omega)$ and phase-frequency $\varphi_\beta(\omega)$ characteristics of the boundary (15) for different values of the parameter β.

$$\mu(\tau) \underset{\beta\to\infty}{\cong} q\left[\frac{\omega\tau_m \exp(-b_1^2\tau/\tau_0)}{1+\omega^2\tau_m^2} + \frac{\sin(\omega\tau - \operatorname{arctg}\omega\tau_m)}{\sqrt{1+\omega^2\tau_m^2}}\right].$$

We now examine the behavior of the boundary for small times.

1. Thermal Boundary. In the interval $0<\tau\ll\tau_0$ we have

$$K(\tau) \cong 1/\sqrt{\pi\tau_0\tau}\ [2]$$

and

$$\mu(\tau) \cong \frac{q}{\sqrt{\pi\tau_0}}\int_0^\tau \frac{\sin\omega\tau'}{\sqrt{\tau-\tau'}}d\tau' = q\sqrt{\frac{2}{\omega\tau_0}}\,[\sin\omega\tau C_2(\omega\tau) - \cos\omega\tau S_2(\omega\tau)].$$

Here $C_2(y)$ and $S_2(y)$ are Fresnel integrals [5]. Expanding these latter for large values of the argument, and being limited to two terms in the expansion, we obtain a simplified formula for the solution in the times $2\pi/\omega \lesssim \tau \ll \tau_0$:

$$\mu(\tau) \cong \frac{q}{\sqrt{\omega\tau_0}}\,[\sin(\omega\tau - \pi/4) + 1/\sqrt{\pi\omega\tau}\,]. \tag{16}$$

2. Phase Boundary. In the interval $0 \leqslant \tau \ll \tau_0 \min(1;\ \beta^2)$ we have $K(\tau) \cong \frac{1}{\tau_m}\left(1 - \frac{2}{\beta}\sqrt{\frac{\tau}{\pi\tau_0}}\right)$ [2].

We hence find

$$\mu(\tau) \cong \frac{q}{\omega\tau_m}\left\{1 - \cos\omega\tau + \frac{1}{\beta}\sqrt{\frac{2}{\omega\tau_0}}\left[\sin\omega\tau S_2(\omega\tau) + \cos\omega\tau C_2(\omega\tau) - \sqrt{\frac{2}{\pi}\omega\tau}\right]\right\}.$$

In the narrower time interval $2\pi/\omega \lesssim \tau \ll \tau_0 \min(1;\ \beta^2/10)$, simplification yields

$$\mu(\tau) \cong q\,\frac{1-\cos\omega\tau}{\omega\tau_m}. \tag{17}$$

From a comparison of (14) and (15) with (16) and (17), respectively, it is possible to assess qualitatively the nature of the effect of a rapidly oscillating thermal flux ($2\pi/\omega \ll \tau_0$). By the lapse of several periods of flux oscillations the boundary starts to perform oscillations close to the steady value but around the shifted middle position which departs to the initial value s_0 with time.

Periodic Perturbation. We assume that

$$q(\tau) = q_0 + \sum_{n=1}^{\infty} q_n \sin(n\omega\tau - \alpha_n).$$

According to the elucidation above, the regular boundary regime will have the form

$$\bar{\mu}(\tau) = q_0 + \sum_{n=1}^{\infty} q_n D_\beta(n\omega)\sin(n\omega\tau - \alpha_n - \varphi_\beta(n\omega)),$$

where the characteristic time of its build-up does not exceed the quantity τ_0/b_1^2 (it agrees with this quantity for $q_0 \neq 0$ or when the series for $q(\tau)$ is infinite).

We turn again to the initial general solution (2)-(5) and examine one case when it can be simplified.

Slow-Small Perturbations. Let the flux variation be small together with its first and second derivatives, more accurately

$$q(0)=0,\ \dot{q}(0)=0,\ \max_{\tau\geqslant 0}|q(\tau)|\to 0,\ \tau_0\max(1;\ \beta)\max_{\tau>0}|\dot{q}(\tau)|\to 0,$$
$$\tau_0^2\max(1;\ \beta^2)\max_{\tau>0}|\ddot{q}(\tau)|\to 0.$$

We write the solution in the new variables [2]: $t = \tau$, $p = x/s(\tau)$, $v(x, \tau) \equiv u(p, t) = T_0 + p(T_m - T_0) + u_1(p, t)$, $u_1(p, t) = w(p, t) - p(T_m - T_0)\mu(t)$. Because of (3)-(5), we have

$$u_1(p,\ t)=(T_m-T_0)\frac{2}{\tau_0}\sum_{n=1}^{\infty}\left(\frac{\sin pb_n}{\sin b_n}-p\right)\frac{\int\limits_0^t\exp\left(-b_n^2\frac{t-t'}{\tau_0}\right)q(t')dt'}{1+\beta+\beta^2b_n^2}. \tag{18}$$

Integrating twice by parts in (18) and evaluating the necessary sums by using limit theorems of operational calculus [4] (we do not present a detailed derivation), the following final expression can be obtained for the temperature

$$u(p,\ t)=T_0+(T_m-T_0)\left[p-\frac{p(1-p^2)}{6}\tau_0\dot{q}(t)+\frac{p(1-p^2)}{6}\left(\beta+\frac{1}{3}+\frac{1-3p^2}{12}\right)\tau_0^2\ddot{q}(\bar{t})\xi\right], \tag{19}$$

where $0\leqslant\xi=\xi_q(p,\ t,\ \beta)\leqslant 1,\ \xi_q(p,\ 0,\ \beta)=0,\ \xi_q(p,\ t,\ \beta)\underset{t\to\infty}{\longrightarrow}1;\ 0\leqslant\bar{t}=\bar{t}_q(p,\ t,\ \beta)\leqslant t.$ We find analogously for the boundary

$$s(t)=s_0\left\{1-q(t)+\left(\beta+\frac{1}{3}\right)\tau_0\dot{q}(t)-\left[\left(\beta+\frac{1}{3}\right)^2+\frac{1}{45}\right]\tau_0^2\ddot{q}(\bar{\bar{t}})\cdot\zeta\right\}, \tag{20}$$

where $0\leqslant\zeta=\zeta_q(t,\ \beta)\leqslant 1;\ \zeta_q(0,\ \beta)=0;\ \zeta_q(t,\ \beta)\underset{t\to\infty}{\longrightarrow}1;\ 0\leqslant\bar{\bar{t}}=\bar{\bar{t}}_q(t,\ \beta)\leqslant t.$ Let us emphasize that the quantities $\bar{t}, \bar{\bar{t}}, \xi, \zeta$ depend on the function q taken for the argument that varies between 0 and t. However, if the second derivative of the flux is so small that the last terms in (19) and (20) can be neglected as compared with the remaining components (we do not refine the specific conditions for this), then the solution (2) at each time will be expressed only in terms of the value of the flux and its derivative at the same instant. In other words, the dependence of the solution on the flux becomes almost local if the later changes sufficiently slowly and smoothly. Therefore, the integral operators in (3) have their own kind of "finite memory." With respect to the boundary operator, this deduction was made qualitatively in [1], however, an error was committed there in obtaining the formulas of type (20).

NOTATION

$v(x, \tau)$, temperature; τ, time; x, space coordinate; $s(\tau)$, trajectory of the moving boundary; a^2, thermal diffusivity; k, heat conduction; c, specific heat; λ, specific heat of transition, and ρ, density.

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